Int. J. Heat Mass Transfer. Vol. 15, pp. 1647–1658. Pergamon Press 1972. Printed in Great Britain

APPLICATION OF ROUGH SURFACES TO HEAT EXCHANGER DESIGN

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(*Received* 12 *August* 1971 *and in revised form* 17 *November* 1971)

Abstract—Equations are developed to define the performance advantage of roughened tubes in heat exchanger design, relative to smooth tubes of equal diameter. Three rough tube applications are presented: 1. To obtain increased heat exchange capacity; 2. To reduce the friction power; and 3. To permit a reduction of heat-transfer surface area. A heat exchanger design procedure is developed for each application and is based on the use of previously developed generalized heat transfer and friction correlations for rough surfaces. The graphical results of example solutions are presented for the case of "repeated-rib" roughness. These graphs show the heat exchanger performance improvements offered by this roughness type, relative to smooth tubes. Because the heat transfer and friction correlations used for the "repeated-rib" roughness can be used to correlate the data of other types of geometrically similar roughness, the design procedure is applicable to other types of roughness. In addition to flow thru tubes, the design procedure and the calculated results are equally applicable to the problem of parallel flow along the outside surfaces of tubes arranged in a tube or rod bundle.

NOMENCLATURE

A, heat transfer surface area;

B, external-to-smooth internal surface area ratio;

D, pipe inside diameter (to base of rib for rough tubes);

e, rib height;

$(e/D)_{20}$, value of e/D from equation (31) for $e^+ = 20$;

e^+, roughness Reynolds number, $e^+ \equiv eu^*/\nu = (e/D)Re\sqrt{(f/2)}$;

ε, heat exchanger thermal effectiveness;

f, friction factor, $\Delta PD\rho/2LG^2$;

$\bar{g}$, $[(f/2St - 1)/\sqrt{(f/2)} + u_e^+]Pr^{-0.57}$ (repeated-ribs);

G, mass velocity (mass flow per unit area);

G^*, G_s/G;

h, heat-transfer coefficient on considered surface;

h_0, heat-transfer coefficient on external surface (for flow in tubes);

K, overall heat conductance $K = hA$ for prescribed heat flux; $K = UA$ for heat exchange between two fluids;

L, length of the flow passage;

m, $\bar{g}\,Pr^{0.57} - u_{e^+}$, from equation (17);

n, $1.07 + 12.7\sqrt{(f_s/2)}(Pr^{\frac{2}{3}} - 1)$, see equation (17);

N, number of tubes in flow passage;

p, distance between repeated-ribs;

P, flow friction power;

Pr, Prandtl number;

Q, heat transfer rate [W];

r, h_s/Bh_0 ratio of smooth internal-to-external surface heat conductance;

Re, Reynolds number, DG/μ;

St, Stanton number;

u_e^+, $\sqrt{(2/f)} + 2.5\ln(2e/D) + 3.75$;

U, overall heat conductance per unit area [W/m^{2}K].

Subscripts

Unsubscripted variables refer to rough surfaces.

Subscript s refers to smooth surface.

INTRODUCTION

Two previous publications [1, 2] have presented generalized friction factor and Stanton number correlations for turbulent flow in tubes, which have a "repeated-rib" roughness. This paper discusses application of these correlations to heat exchanger design. Figure 1 shows a sketch of the roughness geometry and defines the roughness parameters as e/D and p/e. The friction data for geometrically similar roughness

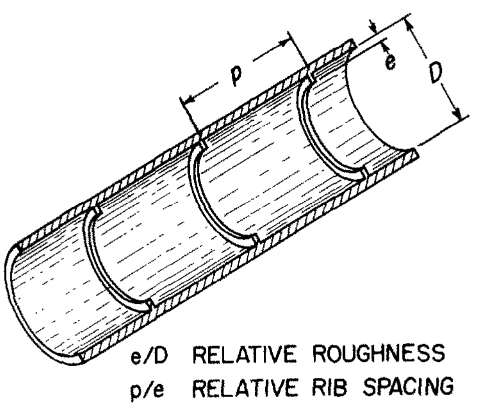

FIG. 1. Sketch of repeated-rib roughness geometry.

(p/e = constant) were correlated by the function u_e^+ vs. e^+, where

$$u_e^+ \equiv \sqrt{(2/f)} + 2{\cdot}5\ \ln(2e/D) + 3{\cdot}75. \qquad (1)$$

The heat-transfer data were correlated by $\bar{g}$ vs. e^+, where

$$\bar{g} \equiv \left(\frac{f/2St - 1}{\sqrt{(f/2)}} + u_e^+\right) Pr^{-0{\cdot}57}. \qquad (2)$$

Graphs of the functions u_e^+ and $\bar{g}$ are given in [1].

This work defines the advantages of repeated-rib roughness, relative to smooth tubes, when used in a heat exchanger having specified flow conditions. For this problem, the smooth and rough tube heat exchanger designs have the same:

A. Tube diameter
B. Mass flow rate
C. Entering fluid temperatures and pressures.

The equations developed here are for roughness on the inner surface of a tube. With proper interpretation, the same equations are applicable to parallel flow in a tube or rod bundle, which has roughness on the outer surface of the tubes. This application will be discussed later.

THE THREE HEAT EXCHANGER APPLICATIONS

There are three applications in which roughened tubes may offer an advantage over smooth tubes:

A. To obtain reduced heat-transfer surface area for equal heat exchange and friction power (e.g. $P/P_s = 1$ and $Q/Q_s = K/K_s = 1$).
B. To obtain increased heat exchange capacity for equal surface area and friction power (e.g. $P/P_s = 1$ and $A/A_s = 1$).
C. To reduce the friction power expenditure for equal heat-transfer surface area (e.g. $A/A_s = 1$ and $Q/Q_s = K/K_s = 1$).

Thus, the three parameters of interest are A/A_s, K/K_s and P/P_s. For each case of interest, the designer specifies two of the three parameters and solves for the third. The solution procedure yields the roughness (e/D) required to meet the specified criteria for arbitrary values of the relative mass velocity $G^* \equiv G_s/G$. By repeating the solution for several values of G^*, the designer may select the particular e/D best suited for his application.

A SPECIAL CASE—PRESCRIBED WALL TEMPERATURE OR HEAT FLUX

The equations which must be satisfied for each of the three application cases is illustrated by considering the special case of prescribed wall temperature or a prescribed heat flux. The heat flux boundary condition is associated with heat generation by electric resistance heating. In these special cases, the relative heat conductance of the rough and smooth tube exchangers is $K/K_s = hA/h_sA_s$ or equivalently:

$$\frac{K}{K_s} = \frac{St}{St_s} \cdot \frac{A}{A_s} \cdot \frac{1}{G^*}. \tag{3}$$

If $K/K_s = 1$, both exchangers have the same thermal effectiveness ($\varepsilon/\varepsilon_s = 1$). If $K/K_s > 1$, the thermal effectiveness of the rough tube design is greater than that of the smooth tube design ($\varepsilon/\varepsilon_s > 1$). Because $\varepsilon/\varepsilon_s > 1$, $Q/Q_s < K/K_s$. Given K_s, K/K_s and the entering flow parameters, Q/Q_s is easily computed from the applicable thermal effectiveness equation or graph. Such equations and graphs are given in heat transfer textbooks; for example, see Chapter 2 of [9].

The relative friction power relation is:

$$\frac{P}{P_s} = \frac{f}{f_s} \frac{A}{A_s} \left(\frac{1}{G^*}\right)^3. \tag{4}$$

Because the heat exchanger mass flow rate and tube diameter are held constant, G^* is changed by varying N/N_s, where N is the number of parallel flow passages. For flow thru tubes, $G^* = Re_s/Re$. The symbol A is the total heat exchange surface area ($A = \pi DNL$).

By eliminating G^* between equations (3) and (4),

$$\frac{K/K_s}{(P/P_s)^{1/3}(A/A_s)^{2/3}} = \frac{St/St_s}{(f/f_s)^{1/3}}. \tag{5}$$

Equation (5) defines an expression containing the three parameters K/K_s, P/P_s, A/A_s in terms of a Stanton number–friction factor ratio. For each of the three application cases, two of these parameters are set at the value one. Then, the remaining parameter is given by equation (5) in terms of a Stanton–friction factor ratio. Table 1 shows the results of equations (3)–(5) when applied to each of the three heat exchanger applications. The third column specifies the constraints required for each case. The fourth column gives the "parameter of interest", which is calculated from equation (5). The last column lists the relative mass velocity (G^*) required to satisfy the constraints, and is calculated from equations (3) and (4). For cases A and C, $Q/Q_s = K/K_s = 1$.

The next step is to use the equations of Table 1 to compute the roughness size required for each case of interest. Assume that Pr and Re_s are known, and that we are interested in case A for flow thru tubes ($G^* = Re_s/Re$). From Table 1,

$$\frac{A}{A_s} = \frac{(f/f_s)^{1/2}}{(St/St_s)^{3/2}} \tag{6}$$

$$G^* = \frac{(f/f_s)^{1/2}}{(St/St_s)^{1/2}}. \tag{7}$$

The rough tube Stanton number and friction factor are assumed to be known functions of

Table 1. Relative performance for rough tubes with specified wall temperature

Case	Description	Constraints	Parameter of interest	Relative mass velocity (G^*)
A	Reduced surface area	$\frac{P}{P_s} = \frac{Q}{Q_s} = 1$	$\frac{A}{A_s} = \frac{(f/f_s)^{1/2}}{(St/St_s)^{3/2}}$	$G^* = \frac{(f/f_s)^{1/2}}{(St/St_s)^{1/2}}$
B	Increased heat transfer	$\frac{P}{P_s} = \frac{A}{A_s} = 1$	$\frac{K}{K_s} = \frac{St/St_s}{(f/f_s)^{1/3}}$	$G^* = (f/f_s)^{1/3}$
C	Reduced friction power	$\frac{Q}{Q_s} = \frac{A}{A_s} = 1$	$\frac{P}{P_s} = \frac{f/f_s}{(St/St_s)^3}$	$G^* = St/St_s$

e/D and Re by virtue of equations (1) and (2). Therefore, the two equations, equations (6) and (7), implicitly contain three variables—A/A_s, G^* and e/D. It is necessary to arbitrarily specify one of these unknowns in order to define a tractable problem. Assume that G^* is specified and that a solution for A/A_s and e/D has been obtained. If the designer wishes to minimize A/A_s, there is no certainty that the G^* initially selected will give the minimum A/A_s. It is necessary to repeat the solution procedure for several values of G^*. In reality, the solution procedure is more difficult than suggested. Equations (1) and (2) cannot be manipulated to give explicit algebraic equations of the form $f(e/D, Re)$ and $St(e/D, Re)$. This complication will be treated later, when the detailed solution procedure is developed.

The equations of Table 1 are equally applicable to parallel flow in "rod bundles", if $Re = Re_s$. This problem is of interest in nuclear reactor design. A number of papers have appeared, e.g. [3–5], in which the equations of Table 1 are applied to the rod bundle problem for cases B and C. The application of these equations to the rod bundle problem will be discussed in a later section.

GENERALIZED HEATING BOUNDARY CONDITIONS

The prescribed wall temperature or heat flux boundary condition treated in the preceding section is a special case of a more general problem. The more general problem involves the transfer of heat, between two fluids, across a pipe wall. We will develop the governing equations for this problem, and define the conditions under which these equations reduce to those in Table 1.

Consider rough and smooth tube heat exchangers which have equal flow rates and equal entering fluid temperatures. These specified temperature and flow conditions imply that $K/K_s = UA/U_sA_s$. By definition

$$\frac{1}{UA} = \frac{1}{hA}\left(1 + \frac{h}{Bh_0}\right) \tag{8}$$

where A is the total internal surface area and B is the ratio of external-to-internal surface area. The heat-transfer coefficients on the inside and outside of the tubes are h and h_0, respectively. With equation (8), the heat transfer equations for the smooth and rough tube exchangers are

$$\frac{1}{U_sA_s} = \frac{1}{h_sA_s}(1 + r) \tag{9}$$

$$\frac{1}{UA} = \frac{1}{hA}\left(1 + r\frac{h}{h_s}\right). \tag{10}$$

The symbol $r \equiv h_s/Bh_0$ is the ratio of the heat conductance on the inside and outside of the tubes in the smooth tube exchanger. Substituting equations (9) and (10) in $K/K_s = UA/U_sA_s$

$$\frac{K}{K_s} = \frac{A}{A_s}\left[\frac{1 + r}{(h_s/h) + r}\right]. \tag{11}$$

Because both heat exchangers have equal flow rate and equal tube diameters, $h/h_s = (St/St_s)/G^*$. With this substitution in equation (11)†

$$\frac{K}{K_s} = \frac{A}{A_s}\left[\frac{1 + r}{G^*(St_s/St) + r}\right]. \tag{12}$$

Equation (12) reduces to equation (3) if $r = 0$. The previously developed friction power equation (4) still applies to the present situation. If G^* is eliminated between equations (4) and (12)

$$\frac{(K/K_s)(A/A_s)^{\frac{1}{3}}}{(P/P_s)^{\frac{1}{3}}[(A/A_s)(1 + r) - r(K/K_s)]} = \frac{St/St_s}{(f/f_s)^{\frac{1}{3}}}. \tag{13}$$

Equation (13) reduces to equation (5) for $r = 0$. The condition $r = 0$ states that all of the thermal resistance is on the inside of the tube and is mathematically equivalent to the prescribed wall temperature or heat flux boundary condition.

Table 2 shows the results of equations (4), (12) and (13) for each of the three heat exchanger applications. By substituting the application constraints (column 3) in equation (12), the results are listed in column 4 of Table 2. The

† Equation (12) does not include the thermal resistance of the pipe wall. This resistance can be included in equation (12) by defining r equal to $h_s/Bh_0 + h_st/k$, where t is the pipe wall thickness and k is the thermal conductivity of the pipe.

Table 2. Relative performance for rough tubes when heat is exchanged between two fluids, across a pipe wall

Case	Description	Constraints	Parameter of interest	Relative mass velocity G^*
A	Reduced surface area	$\frac{P}{P_s} = \frac{Q}{Q_s} = 1$	$\frac{A}{A_s} = \frac{G^*(St_s/St) + r}{1 + r}$	$\frac{f/St}{f_s/St_s} = (G^*)^2(1 + r) - \frac{r(f/f_s)}{G^*}$
B	Increased heat transfer	$\frac{P}{P_s} = \frac{A}{A_s} = 1$	$\frac{K}{K_s} = \frac{(1 + r)}{(f/f_s)^{\frac{1}{3}}/(St/St_s) + r}$	$G^* = \left(\frac{f}{f_s}\right)^{\frac{1}{3}}$
C	Reduced friction power	$\frac{Q}{Q_s} = \frac{A}{A_s} = 1$	$\frac{P}{P_s} = (f/f_s)/(St/St_s)^3$	$G^* = \frac{St}{St_s}$

last column lists the relative mass velocity (G^*) required to satisfy the constraints. This G^* is calculated from equations (4) and (12). Table 2 gives an implicit equation for G^* in case A. All equations of Table 2 reduce to those of Table 1 for $r = 0$.

SOLUTION PROCEDURE

Given Re_s, Pr and r we need to solve the equations of Table 2 for each case of interest. The procedure adopted is to arbitrarily specify G^* and solve for the St, f and e/D which satisfy the Table 2 equations. The solution procedure is outlined for each case.

Case A (Reduced surface area)

Table 2 gives

$$\frac{f/St}{f_s/St_s} = (G^*)^2(1 + r) - r\frac{(f/f_s)}{G^*}. \tag{14}$$

By equation (2),

$$\frac{f}{St} = 1 + \sqrt{(f/2)}(\bar{g}\,Pr^{0\cdot57} - u_e^+). \tag{15}$$

The heat-transfer equation recommended for smooth tubes is due to Petukhov [6, 7]

$$\frac{f_s}{St_s} = 1{\cdot}07 + 12{\cdot}7\sqrt{(f_s/2)}(Pr^{\frac{2}{3}} - 1). \tag{16}$$

Substituting equations (15) and (16) into (14)

$$\frac{1 + \sqrt{(f/2)}(\bar{g}\,Pr^{0\cdot57} - u_e^+)}{1{\cdot}07 + 12{\cdot}7\sqrt{(f_s/2)}(Pr^{\frac{2}{3}} - 1)}$$

$$= (G^*)^2(1 + r) - r\frac{(f/f_s)}{G^*}. \tag{17}$$

Equation (17) is to be solved for the rough tube friction factor. For simplicity, let $(\bar{g}\,Pr^{0\cdot57} - u_e^+) \equiv m$ and $1{\cdot}07 + 12{\cdot}7\sqrt{(f_s/2)}\,(Pr^{2/3} - 1) \equiv n$. Writing equation (17) in the quadratic form

$$\frac{2r}{f_s G^*}[\sqrt{(f/2)}]^2 + \frac{m}{n}\sqrt{(f/2)} + \left[\frac{1}{n} - (G^*)^2(1 + r)\right] = 0. \tag{18}$$

The solution of equation (18) for $\sqrt{(f/2)}$ is

$$\sqrt{(f/2)} = \frac{f_s m G^*}{4nr}\left[\!\left[-1 + \sqrt{\left\{1 - \left[\frac{1}{n} - (G^*)^2(1 + r)\right]\frac{8n^2 r}{f_s m^2 G^*}\right\}}\right]\!\right]. \tag{19}$$

Because Re_s, Pr, r and G^* are known, the only unknown on the right side of equation (19) is $m \equiv g\ Pr^{0\cdot57} - u_e^+$; this is a function of the roughness Reynolds number, e^+. Since equation (19) contains the two unknowns, $\sqrt{(f/2)}$ and e^+, a second equation relating these variables is necessary. This second equation is provided by the equation which defines e^+:

$$\sqrt{(f/2)} = \frac{e^+}{(e/D)Re}. \tag{20}$$

The variable e/D is eliminated from equation (20) using equation (1). Solving equation (1) for e/D

$$e/D = \frac{1}{2}\exp\left[\frac{u_e^+ - \sqrt{(2/f)} - 3{\cdot}75}{2{\cdot}5}\right]. \quad (21)$$

Substitution of equation (21) in (20) gives

$$\sqrt{(f/2)} = \frac{0{\cdot}5\, e^+}{Re\exp\left[\frac{\sqrt{(2/f)} + 3{\cdot}75 - u_e^+}{2{\cdot}5}\right]}. \quad (22)$$

The simultaneous solution of equations (19) and (22) gives $\sqrt{(f/2)}$ and e^+. These equations are easily solved on a digital computer, using an iterative procedure.

The remaining steps in the solution are easily performed using the calculated values of $\sqrt{(f/2)}$ and e^+. These steps are:

1. Compute e/D from equation (21).
2. The rough tube Stanton number, calculated from equation (1) is

$$St = \frac{f/2}{1 + \sqrt{(f/2)}(\bar{g}\,Pr^{0{\cdot}57} - u_e^+)}. \quad (23)$$

3. The relative heat-transfer surface area is calculated from the A/A_s equation of Table 2 for case A.

$$\frac{A}{A_s} = \frac{(St_s/St)\,G^* + r}{1 + r}. \quad (24)$$

For an arbitrary value of G^*, we have determined the specific roughness size (e/D) which yields $P/P_s = Q/Q_s = 1$, and the relative surface area requirement (A/A_s). It is expected that the designer would repeat the solution procedure for different values of G^*. Then a graph of A/A_s vs. G^* would allow the designer to select a specific roughness size, based on the combined considerations of material savings (A/A_s) and the relative heat exchanger frontal area (G^*).

Case B (Increased heat transfer)

The solution for K/K_s is easily obtained for arbitrary G^*. The steps are:

1. Compute f from the equation for G^* (case B Table 2)

$$f = f_s(G^*)^{1/3}. \quad (25)$$

2. Using the known f, solve for e^+ using equation (22). An iterative process is required.
3. Compute e/D from equation (21).
4. The rough tube Stanton number is given by equation (23).
5. K/K_s is then computed by the equation for K/K_s in Table 2 for case B.
6. Given K and K_s, Q/Q_s is computed using the heat exchanger thermal effectiveness relations given in heat-transfer textbooks [9].

Case C (Reduced friction power)

The steps in the solution for P/P_s for arbitrary G^* are:

1. Compute the required rough tube Stanton number from the equation for G^* (case C of Table 2); e.g.

$$St = St_s G^*. \quad (26)$$

2. Equation (23) solved for $\sqrt{(f/2)}$ is

$$\sqrt{(f/2)} = \frac{St\, m}{2}\left[1 + \sqrt{\left(1 + \frac{4}{St\, m^2}\right)}\right]. \quad (27)$$

Because $m \equiv \bar{g}\ Pr^{0{\cdot}57} - u_e^+$ is a function of e^+, a second equation relating $\sqrt{(f/2)}$ and e^+ is needed. Equation (22) provides the necessary second equation. The simultaneous, iterative solutions of equations (22) and (27) yield $\sqrt{(f/2)}$ and e^+.

3. e/D is then computed from equation (21).
4. The rough tube Stanton number is given by equation (23).
5. The relative friction power is given by the equation for P/P_s from Table 2, case C.

$$\frac{P}{P_s} = \frac{f/f_s}{(St/St_s)^3}. \quad (28)$$

EXAMPLE SOLUTIONS

The equations for the three application cases have been solved on a digital computer for a wide range of Pr, Re_s, G^* and r. The solutions were computed for the repeated-rib roughness geometry with $p/e = 10$, using the correlations presented in [1].

Case A (Reduced surface area)

Figures 2–4 show the graphical results for Case A ($Q/Q_s = P/P_s = 1$) in the form A/A_s vs. G^*, for $Pr = 1$, 10 and 100, respectively. Each figure shows the effect of the parameter $r \equiv h_s/Bh_0$ on the required surface area. The value of $r = 0$ corresponds to the prescribed wall temperature or heat flux boundary condition. The greatest surface area reduction is associated with the $r = 0$ condition. The upper curve of each figure is for $r = 2{\cdot}0$, which states that the thermal resistance on the inside of the smooth tube is only one-third of the total. Thus, if this smooth tube is replaced by a rough surface having a 100 per cent larger heat transfer coefficient, the total thermal resistance would be reduced only 17 per cent. The figures do not show the e/D associated with each value of A/A_s, but these are easily determined from equation (21) of the solution procedure.

Lines of constant e^+ are also shown on

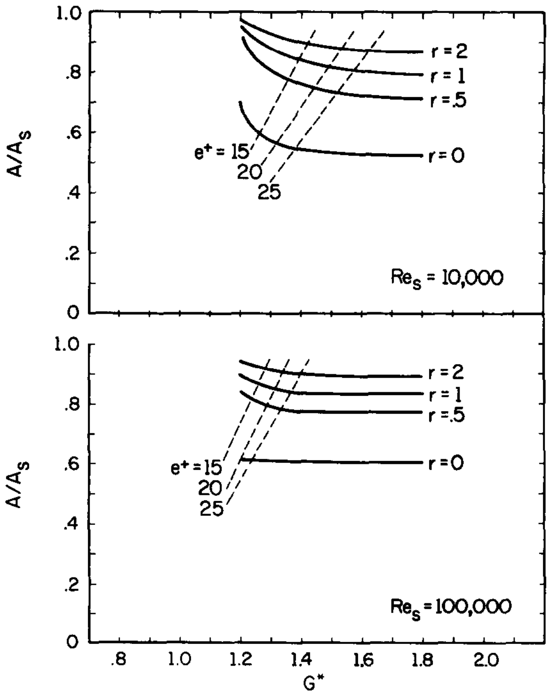

FIG. 2. A/A_s vs. G^* for $Pr = 1$, with $P/P_s = Q/Q_s = 1$. Repeated-rib roughness ($p/e = 10$).

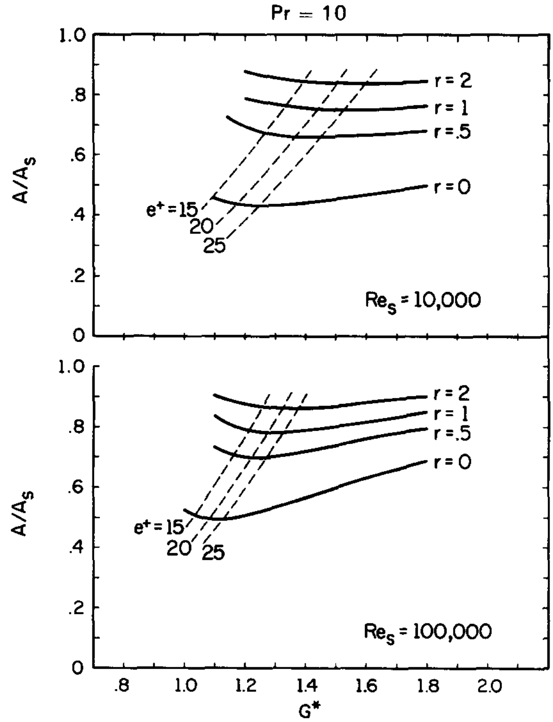

FIG. 3. A/A_s vs. G^* for $Pr = 10$, with $P/P_s = Q/Q_s = 1$. Repeated-rib roughness ($p/e = 10$).

Figs. 2–4. A minimum A/A_s occurs when $e^+ \simeq 20$, for $Pr = 10$ and 100. Although a minimum A/A_s is not observed for $Pr = 1$, the values of A/A_s at $e^+ = 20$ are no more than 5 per cent greater than the smallest A/A_s shown on Fig. 2. Therefore, the specification $e^+ = 20$ is a good approximate design specification when the designer wishes to attain minimum A/A_s.

The designer would normally select the

Table 3. Relative frontal area of rough tube design to smooth tube design ($Re_s = 100000$ at $e^+ = 20$)

	r		
Pr	0	0·5	1·0
1	1·20	1·28	1·32
10	1·08	1·22	1·28
100	0·95	1·16	1·23

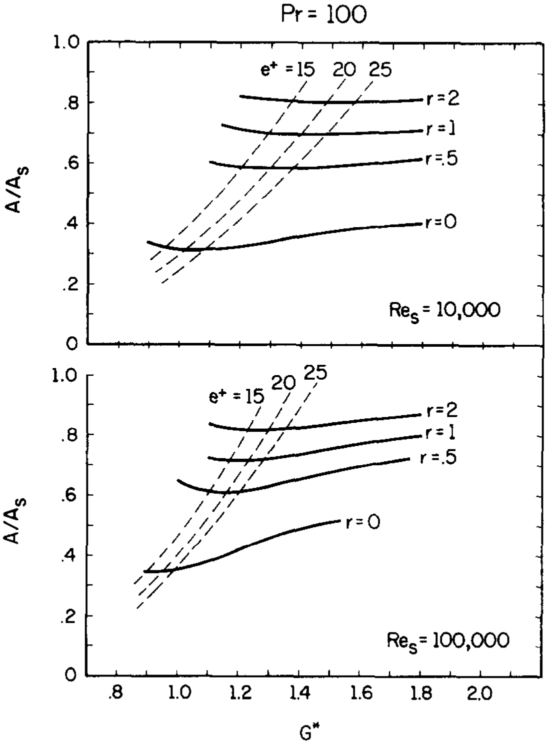

FIG. 4. A/A_s vs. G^* for $Pr = 100$, with $P/P_s = Q/Q_s = 1$. Repeated-rib roughness ($p/e = 10$).

roughness size which gives minimum A/A_s, providing the heat exchanger frontal area is within acceptable limits. Table 3 lists the relative frontal area (G^*) of the rough and smooth tube designs for several values of r at $Re_s = 100000$ and $e^+ = 20$.

Case B (Increased heat transfer)

Figures 5–7 show the calculated results for case B ($P/P_s = A/A_s = 1$) in the form K/K_s vs. G^*, for $Pr = 1$, 10 and 100, respectively. The figures contain curves of constant r to show the effect of the external thermal resistance on the heat exchanger capacity increase. The greatest capacity increase occurs when $r = 0$. Values of e^+ corresponding to G^* are noted above the upper curves on each figure. Except for $Pr = 1$, the maximum K/K_s occurs at $e^+ \simeq 20$, as was observed for Figs. 3 and 4. The value of K/K_s at $e^+ = 20$ for $Pr = 1$ (Fig. 5) is within 7·5 per cent of the maximum K/K_s. Therefore, the specification $e^+ = 20$ again appears to be a reasonable design specification for attainment of K/K_s near its maximum value.

Case C (Reduced pumping power)

The results for case C ($Q/Q_s = A/A_s = 1$) are shown on Fig. 8, by the graphs of P/P_s vs. G^*. The results are independent of the parameter r, since $Q/Q_s = 1$ requires $h = h_s$ for all values of r. For $Pr = 1$, P/P_s decreases with increasing G^*, and $P/P_s \simeq 0·36$ at $G^* = 2$. The curves for $Pr = 10$ are limited in the direction of smaller G^* by the requirement $e^+ > 6$. This requirement exists because the heat transfer and friction correlations, equations (1) and (2), are based on data for $e^+ > 6$. No curves are shown for $Pr = 100$, because values of $G^* < 2$ would require $e^+ < 6$. Significant friction power re-

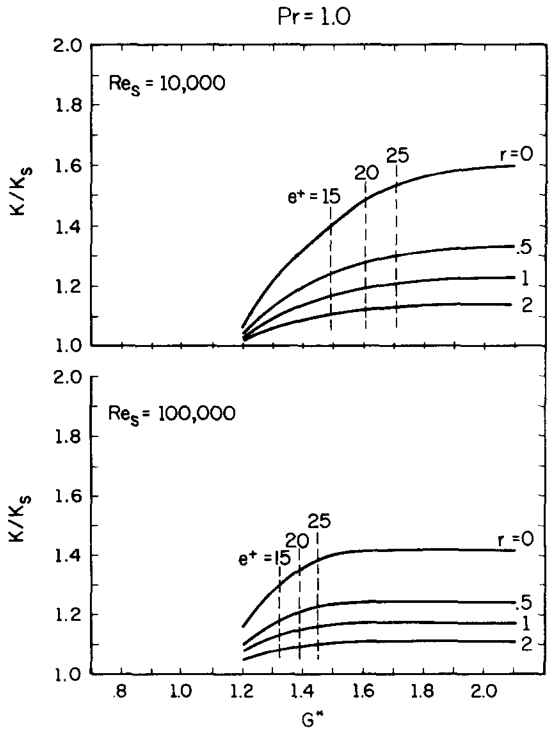

FIG. 5. K/K_s vs. G^* for $Pr = 1$, with $P/P_s = A/A_s = 1$. Repeated-rib roughness ($p/e = 10$).

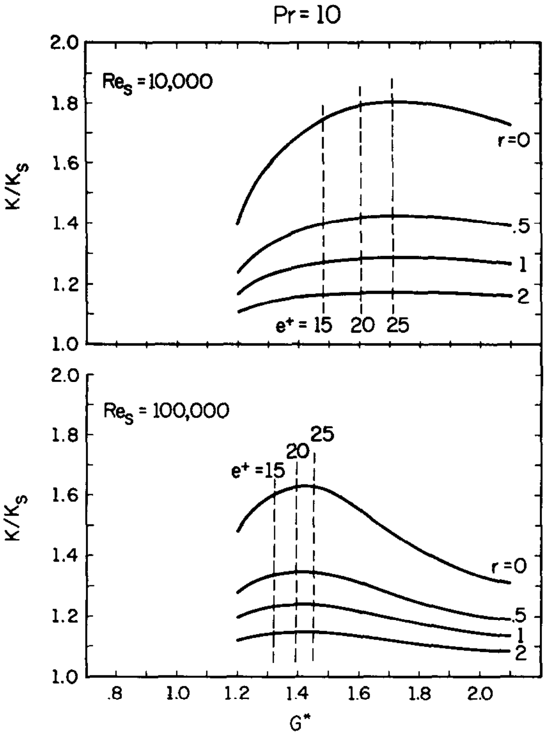

FIG. 6. K/K_s vs. G^* for $Pr = 10$, with $P/P_s = A/A_s = 1$. Repeated-rib roughness ($p/e = 10$).

duction would be yielded by very small values of e/D for high Prandtl number fluids.

SIMPLIFIED SOLUTION PROCEDURE

We have previously noted that the condition $e^+ = 20$ will yield A/A_s near its minimum values for case A, and K/K_s near its maximum value for case B. If the condition $e^+ = 20$ is accepted as a good design specification, it is possible to develop a simplified solution procedure. This procedure permits direct calculation of St, f and e/D, thus avoiding the iterative solution process.

At $e^+ = 20$, [1] gives $\bar{g} = 11{\cdot}0$ and $u_e^+ = 4{\cdot}2$ for repeated-rib roughness with $p/e = 10$. Substituting $u_e^+ = 4{\cdot}2$ in equation (1), and solving for $\sqrt{(2/f)}$

$$\sqrt{(2/f)} = 0{\cdot}45 - 2{\cdot}5\ln(2e/D). \quad (29)$$

By definition $e^+ \equiv (e/D)\,Re\,\sqrt{(f/2)}$. Let $(e/D)_{20}$ be the value of e/D that satisfies this equation when $e^+ = 20$. Then

$$(e/D)_{20} = \frac{20}{Re}\sqrt{(2/f)}. \quad (30)$$

Substitution of equation (29) in (30) gives

$$(e/D)_{20} = \frac{20}{Re}\{0{\cdot}45 - 2{\cdot}5\ln[2(e/D)_{20}]\}. \quad (31)$$

Figure 9 shows the solution of equation (31), plotted as $(e/D)_{20}$ vs. Re. A curve fit of Fig. 9 gives the approximate empirical equation for $10^4 < Re < 10^6$.

$$(e/D)_{20} \cong 37{\cdot}4/Re^{0{\cdot}83}. \quad (32)$$

The steps in the simplified solution procedure, based on $e^+ = 20$ for arbitrary Re are:

1. At the selected Re, read $(e/D)_{20}$ from Fig. 9, or compute $(e/D)_{20}$ using equation (32).

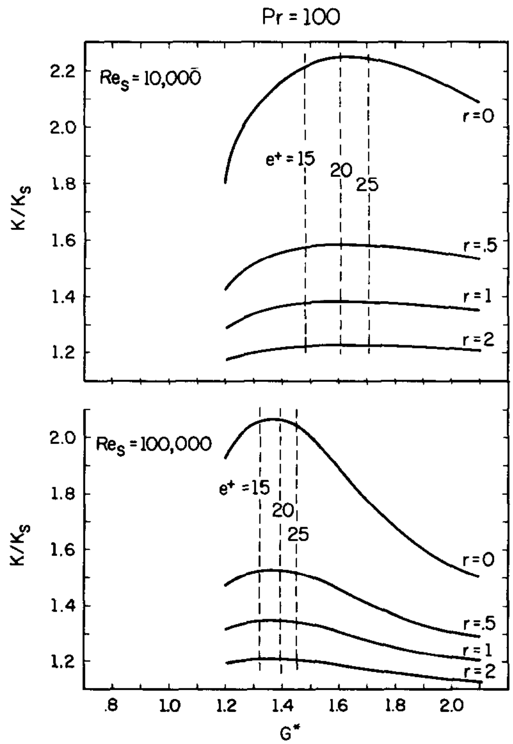

FIG. 7. K/K_s vs. G^* for $Pr = 100$, with $P/P_s = A/A_s = 1$. Repeated-rib roughness ($p/e = 10$).

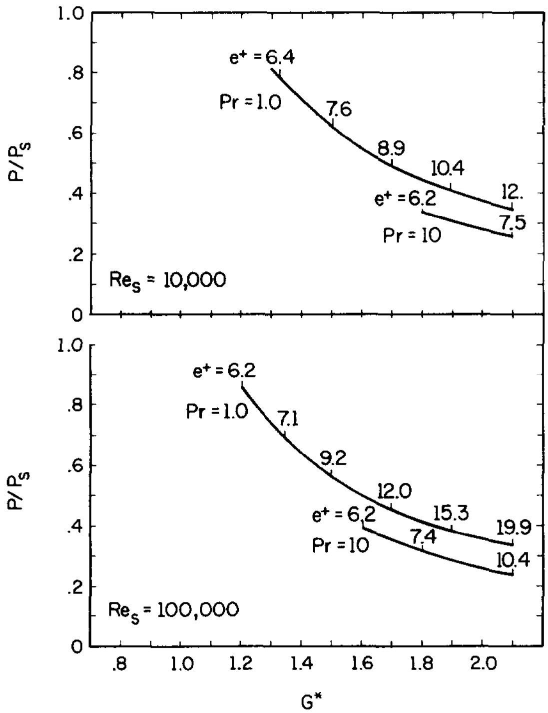

FIG. 8. P/P_s vs. G^* for all Pr, with $Q/Q_s = A/A_s = 1$. Repeated rib roughness ($p/e = 10$).

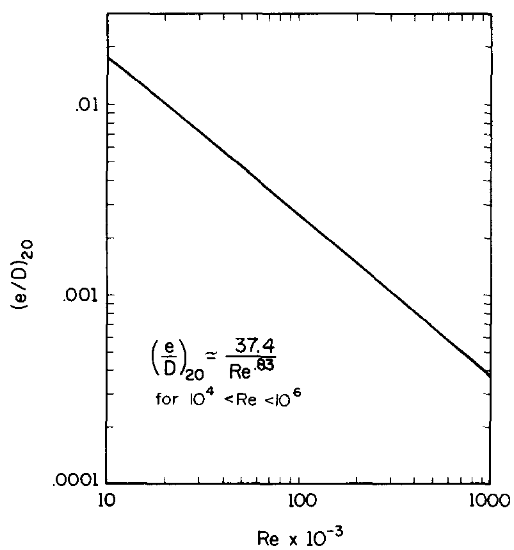

FIG. 9. Graph of solution of equation (31) for repeated-rib roughness ($p/e = 10$) and $e = 20$.

2. Calculate $\sqrt{(2/f)}$ from equation (29).
3. The Stanton number is computed from equation (1) with $\bar{g} = 11$ and $u_e^+ = 4{\cdot}2$.

This equation solved for the Stanton number is

$$St = \frac{f/2}{1 + \sqrt{(f/2)}\,(11\,Pr^{0{\cdot}57} - 4{\cdot}2)}. \quad (33)$$

GENERALITY OF THE SOLUTION PROCEDURE

The solution procedure is applicable to any flow geometry for which the hydraulic diameter applies. In addition to flow inside tubes, which we have discussed in detail, there is a second important flow geometry. This is parallel flow along the outside surface of tubes arranged in a tube or rod bundle. We have already referred to the problem of flow in rod bundles which is of interest to nuclear reactor designers; for this case, $r = 0$. The equations are equally applicable to parallel flow along the outer surface of tubes arranged in a bundle. When a second fluid passes thru the tubes in the bundle, $r > 0$. For these applications, the previously developed equations apply if the smooth and rough tube bundles have an equal number of tubes of the same diameter. Then the relative mass velocity (G^*) is changed by varying the spacing between the tubes or rods. Because the total mass flow rate is constant and the hydraulic diameter is not a function of the spacing between rods, $Re_s/Re = 1$. For a two-fluid heat exchanger, the parameter r is interpreted as the heat conductance ratio (external surface-to-internal surface) of the smooth external tube design. Also, B is the ratio of the internal-to-external surface areas per unit length for the smooth external surface heat exchanger, and h_0 is the heat transfer coefficient on the inside surface of these tubes. The results of the example solutions shown by Figs. 2–8 are applicable to the problem of parallel flow in tube or rod bundles.

Although the solution procedure has been illustrated for the repeated-rib roughness the same equations can be utilized for other roughness geometries. Equations (1) and (2) are applicable to other types of geometrically similar roughness. However, different curves of $\bar{g}$ and u_e^+ vs. e^+ will be obtained for different roughness types. For example, the solution procedure can be applied to the sand-grain roughness data of Dipprey and Sabersky [8], who also use equations (1) and (2) to correlate their data. Also, the correlations given in [1] can be used to calculate the performance of the repeated-rib roughness having values of p/e greater than 10.

CONCLUSIONS

1. Three applications for rough surfaces in heat exchanger design have been defined. Equations are presented which define the advantage afforded by the rough surface design, relative to a smooth surface design, for the same flow conditions.
2. A procedure is outlined for solving the governing equations for each application case. The procedure utilizes generalized rough surface heat transfer and friction correlations. Therefore, the solution procedure is applicable to other types of geometrically similar roughness, for which the specific correlations have been obtained.
3. The solution procedure is applicable to all flow geometries for which the hydraulic diameter concept applies. It is also applicable to a wide range of heating boundary conditions.
4. Example solutions are given for the repeated-roughness geometry with $p/e = 10$. A simplified solution procedure is also given for this roughness geometry.

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APPLICATION DES SURFACES RUGUEUSES A LA CONCEPTION D'ECHANGEUR DE CHALEUR

Résumé—On a établi des équations pour définir le gain de performance des tubes rugueux dans la conception d'échangeurs de chaleur, par rapport à des tubes lisses de même diamètre. Trois applications de tube rugueux sont présentées: 1—pour obtenir une capacité d'échange de chaleur accrue, 2—pour réduire la puissance perdue par frottement, 3—pour permettre une réduction de la surface d'échange thermique. Une procédure de calcul d'échangeur de chaleur est développée pour chaque application et est basée sur l'utilisation de relations générales sur le transfert thermique et le frottement précédemment établies pour des surfaces rugueuses. Les résultats graphiques des solutions données en exemple sont présentés pour le cas de rugosité par annelures régulièrement espacées. Ces graphes montrent les améliorations des performances de l'échangeur de chaleur offertes par ce type de rugosité par rapport à des tubes lisses. Parce que les corrélations de transfert thermique et de frottement utilisées pour une rugosité à annelures régulières peuvent être utilisées pour d'autres types de rugosité géométriquement semblables, la procédure de calcul est applicable à d'autres types de rugosité. En plus de l'écoulement dans les tubes, la procédure et les résultats calculés sont également applicables au problème de l'écoulement parallèle le long des surfaces externes de tubes ou de barreaux disposés en grappe.

ANWENDUNG RAUHER OBERFLÄCHEN BEIM ENTWURF VON WÄRMETAUSCHERN.

Zusammenfassung—Es werden Gleichungen abgeleitet, die das günstigere Übertragungsverhalten von rauhen Rohren beim Entwurf von Wärmetauschern gegenüber glatten Rohren gleichen Durchmessers erkennen lassen. Drei Anwendungsfälle für rauhe Rohre werden besprochen: 1. Erzielung gesteigerter Wärmeaustauschkapazität: 2. Reduktion der Reibungsarbeit: 3. Verringerung der Wärmeaustauschfläche. Für jeden Anwendungsfall wird ein Wärmetauscher—Entwurfsverfahren entwickelt, wobei kürzlich hergeleitete, allgemeine Korrelationen zwischen Wärmetransport und Reibung bei rauhen Oberflächen zugrunde gelegt werden.

Die graphischen Ergebnisse der Beispiellösungen sind für den Rauhigkeitstyp der "wiederholten Rippe" dargestellt. Diese Diagramme zeigen die erzielte Verbesserung im Übertragungsverhalten von Wärmetauschern bei Verwendung des erwähnten Rauhigkeitstyps im Verhältnis zu glatten Rohren. Da die für den Rauhigkeitstyp der "wiederholten Rippe" gewonnenen Korrelation zwischen Wärmetransport und Reibung dazu benutzt werden können, die Daten anderer Typen von geometrisch ähnlichen Rauhigkeiten zu korrelieren, ist das Entwurfverfahren auf andere Typen von Rauhigkeiten anwendbar. Zusätzlich zur Rohrströmung sind das Entwurfsverfahren und die berechneten Ergebnisse gleichermassen anwendbar auf das Problem der Parallelströmung über die Aussenseite von Rohren, die in einem Rohr oder einem Stabbündel angeordnet sind.

ИСПОЛЬЗОВАНИЕ ШЕРОХОВАТЫХ ПОВЕРХНОСТЕЙ ПРИ РАСЧЕТЕ ТЕПЛООБМЕННИКА

Аннотация—Выведены уравнения для определения преимуществ шероховатых труб при расчете теплообменников по сравнению с гладкими трубами равного диаметра. Представлены три случая применения шероховатых труб: I/для увеличения производительности теплообменника; 2/для снижения энергии трения и 3/для уменьшения поверхности теплообмена. Разработана методика расчета теплообменника для каждого случая, которая основана на использовании ранее выведенных обобщенных корреляций теплообмена и трения для шероховатых поверхностей. Представлены графические результаты примеров решений для случая шероховатости типа «повторяющихся ребер». Эти графики показывают улучшение характеристик теплообменника, благодаря введению шероховатостей этого типа по сравнению с гладкими трубами. Так как корреляции, используемые для шероховатости типа «повторяющихся ребер», могут быть применены для корреляции данных при других видах геометрически подобной шероховатости, методика расчета также применима к другим видам шероховатости. Рассмотренная методика для расчета течения в трубах одинаково применима к задаче параллельного течения по внешним поверхностях труб или стержней, собранных в пучок.